

Implementation of the left-right symmetric model in FeynRules/CalcHep

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Abstract

We present an implementation of the manifest left-right symmetric model in FeynRules/CalcHep. The different aspects of the model are briefly described alongside the corresponding elements of the model file. The model file is validated and can be easily translated also to other Feynman diagram calculators, such as MadGraph, Sherpa, etc. The implementation of the left-right symmetric model in FeynRules/CalcHep is a useful step for studying new physics signals with the data generated at the LHC.

Keywords: left-right model; Feynman diagrams; CalcHep model;

PROGRAM SUMMARY

Manuscript Title: Implementation of the left-right symmetric model in FeynRules/CalcHep

Authors: Aviad Roitgrund, Gad Eilam, Shaouly Bar-Shalom

Program Title: Manifest left-right symmetric model

Program location: Available from

https://drive.google.com/folderview?id=0BxMAGX_Tlpi9XORUZw9tS2RaQ0E&usp=sharing.

The URL includes FeynRules model file + translated CalcHep model files.

Programming language: Mathematica

Computer: PC, Unix Workstations

Operating system: Unix

Keywords: left-right model, Feynman diagrams, CalcHep, FeynRules

Classification: 4.4 Feynman diagrams, 5 Computer Algebra.

Nature of problem: 1. Implementing new models of particle interactions. 2. Generating Feynman diagrams for a physical process in the implemented left-right model.

Solution method: Generation of a model file as input for Feynman diagram calculators.

Running time: Approximately 5 minutes to output the Feynman rules in Mathematica (version 7.0.0). Approximately 40 minutes to translate the model to CalcHep with FeynRules 2.0 on a E5400 Pentium Dual-Core desktop. Approximately 20 minutes to translate the model to CalcHep on FeynRules 1.6 on a i5 Quad-Core laptop.

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1. introduction

The main goal of the LHC is to search for signals of new physics beyond the Standard Model (SM), motivated by the shortcomings of the SM. In particular, The SM is incapable of explaining a number of fundamental issues, such as the hierarchy problem (resulting from the large difference between the weak force and the gravitational force), dark matter, the number of families in the quark and lepton sector, etc. It is, therefore, widely believed that new physics beyond the SM will be discovered in the coming years. Among the possible attractive platforms for new physics are left-right symmetric models (LRSM)[1, 2], on which we focus in this paper. In particular, we describe here the implementation of the manifest (and quasi-manifest) left-right symmetric model (MLRSM or QMLRSM, respectively) in **FeynRules/CalcChep**.

These type of LRSM address two specific difficulties of the SM: (i) Parity violation in the weak interactions, and (ii) non-zero neutrino masses implied by the experimental evidence of neutrino oscillation [3]. In particular, the left-right symmetry which underlies LRSM restores Parity symmetry at energies appreciably higher than the electroweak (EW) scale, resulting in the addition of three new heavy gauge bosons, W_2^\pm and Z_2 . Furthermore, in LRSM the neutrinos are massive, where their nature (i.e., whether they are of Majorana or Dirac type) depends on the details of the LRSM.

Early constructions of the LRSM, comprises a Higgs sector with a Higgs bidoublet and two Higgs doublets [1]. In such a setup, the neutrinos are of Dirac type and no natural explanation for their small masses is provided. A later version, the above mentioned MLRSM, incorporates a Higgs bidoublet and two Higgs triplets, which leads to Majorana type neutrinos [2]. In particular, the MLRSM provides a natural setup for the smallness of neutrino (Majorana) masses, relating their mass scale to the large left-right symmetry breaking scale through the see-saw mechanism [4], so that, as opposed to previous work [5], in our implemented model the neutrinos are defined as Majorana particles.

An additional new feature of the implemented model is the user's ability to choose between two types of a LRSM: the manifest LRSM (MLRSM), where diagonalization of the fermion mass matrices yields only positive mass entries, and the quasi-manifest (QMLRSM), where negative mass entries appear after diagonalization.¹

We will describe in brief the MLRSM (and QMLRSM) alongside the model file which implements it. The implementation is based on the MLRSM Lagrangian presented in Ref.[6]² and so a similar notation was used. The model file is generated with the **FeynRules** package [7] (which is a **Mathematica** [8] package), and its output is the computed Feynman rules of the model. We chose the **FeynRules** package as it enabled us to methodically build a model where we can a-priori define its ingredients (i.e., the underlying symmetries, gauge fields, mixing angles and Higgs fields). This enables the **FeynRules** program to derive all the specific vertices of the Lagrangian

¹The formal distinction between the MLRSM and the QMLRSM will be given in Eqs. (7) and (8).

²Ref.[1, 2, 4, 6, 10] carry a detailed description of the LRSM, and in particular of the MLRSM.

which arise from just a few general terms: the interaction terms in the gauge-fermion, gauge-gauge and Yukawa sectors as well as the kinetic and the scalar potential terms. This model building process is, therefore, more complete and has less error making potential in comparison to ad-hoc building of a model where each vertex is added separately. The **FeynRules** package enables the user to translate the Lagrangian of the model to a selection of Feynman diagram calculators, such as **CalcHep**, **Madgraph**, **FeynArts/FormCalc** etc., where the translation can be carried out using a single command aimed to generate the relevant calculator model files [7].³ The model file created in this work has been translated to the Feynman diagram calculator **CalcHep**[9]. It was validated and uploaded to URL https://drive.google.com/folderview?id=OBxMAGX_Tlpi9XORUZW9tS2RaQOE&usp=sharing.

The paper is organized as follows: Sections 2 and 3 describe briefly the Gauge and the Higgs sectors of the LRSM Lagrangian, respectively. Alongside the model description is an explanation of the relevant sections and adjustable parameters of the **FeynRules** model file. In section 4 we validate the model file, and in section 5 we summarize. The list of particles and parameters of the model along with the corresponding model file notations is given in Appendix A. This list is followed by the list of the user adjustable parameters of the model file. In Appendix B we list some useful relations between the model input parameters and the physical parameters and in Appendix C we list the values of the adjustable parameters which were used as input for the validation process.

2. The gauge-fermion sector of the MLRSM/QMLRSM

The LRSM is based on the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, under which the fermion doublets have the irreducible representations shown in Table 1.

The gauge sector of the model consists of two parts. The first is the following gauge-fermion interaction terms

$$\begin{aligned}
L_f &= i \sum \bar{\psi} \gamma^\mu D_\mu \psi \\
&= \bar{L}_L \gamma^\mu \left(i \partial_\mu + g_L \frac{\vec{\tau}}{2} \cdot \vec{W}_{L\mu} - \frac{g'}{2} B_\mu \right) L_L \\
&\quad + \bar{L}_R \gamma^\mu \left(i \partial_\mu + g_L \frac{\vec{\tau}}{2} \cdot \vec{W}_{R\mu} - \frac{g'}{2} B_\mu \right) L_R \\
&\quad + \bar{Q}_L^\alpha \gamma^\mu \left[\left(i \partial_\mu + g_L \frac{\vec{\tau}}{2} \cdot \vec{W}_{L\mu} - \frac{g'}{6} B_\mu \right) \delta_{\alpha\beta} - \frac{g_s}{2} \lambda_{\alpha\beta} \cdot G_\mu \right] Q_L^\beta \\
&\quad + \bar{Q}_R^\alpha \gamma^\mu \left[\left(i \partial_\mu + g_R \frac{\vec{\tau}}{2} \cdot \vec{W}_{R\mu} - \frac{g'}{6} B_\mu \right) \delta_{\alpha\beta} - \frac{g_s}{2} \lambda_{\alpha\beta} \cdot G_\mu \right] Q_R^\beta, \tag{1}
\end{aligned}$$

³For example, translation to CalcHep is carried out using the Mathematica command `WriteCHOutput[LMLRSM]` (see Ref.[7]).

Fermionic fields	content	$SU(3)_C$	\times	$SU(2)_L$	\times	$SU(2)_R$	\times	$U(1)_{B-L}$
L_{iL}	$\begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_L$	1		2		1		-1
L_{iR}	$\begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_R$	1		1		2		-1
Q_{iL}	$\begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_L$	3		2		1		$\frac{1}{3}$
Q_{iR}	$\begin{pmatrix} \nu'_i \\ l'_i \end{pmatrix}_R$	3		1		2		$\frac{1}{3}$

Table 1: The quantum numbers of the fermion fields in the irreducible representations of the LRSM. The index $i = 1, 2, 3$ runs over the number of generations and the ' denotes that these are gauge eigenstates.

where in addition to the eight gluon fields G (also present in the SM), seven gauge fields, $\vec{W}_{L,R}$ and B , are introduced in order to obtain gauge invariance. The appropriate gauge coupling constants are g_s , $g_{L,R}$ and $g' = g_{B-L}$, respectively. The requirement that the Lagrangian is invariant under the left-right symmetry:

$$L_L \leftrightarrow L_R, \quad Q_L \leftrightarrow Q_R, \quad \vec{W}_L \leftrightarrow \vec{W}_R, \quad (2)$$

leads to

$$g_L = g_R. \quad (3)$$

The second part consists of the gauge kinetic terms

$$L_g = -\frac{1}{4}G_a^{\mu\nu}G_{a\mu\nu} - \frac{1}{4}W_{Li}^{\mu\nu}W_{Li\mu\nu} - \frac{1}{4}W_{Ri}^{\mu\nu}W_{Ri\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}, \quad (4)$$

where $G_{\mu\nu}^a$, $W_{L,R\mu\nu}^i$ and $B_{\mu\nu}$ are the field strength tensors of the $SU(3)_C$, $SU(2)_{L,R}$ gauge fields and the $U(1)_{B-L}$ gauge field, respectively.

The relevant adjustable parameters which appear in the gauge-fermion sector of the **FeynRules** model file are the fermion mixing matrices.⁴ These matrices are products of unitary matrices which rotate the fermion fields into their mass eigenstates. The quark fields rotations are

$$\text{Fermion fields} \left\{ \begin{array}{l} U'_{L,R} = V_{L,R}^U U_{L,R} \\ D'_{L,R} = V_{L,R}^D D_{L,R} \end{array} \right\} \text{Mass Eigenstates} \quad (5)$$

⁴The fermion mixing matrices appear also in the Higgs sector, but we chose for convenience to describe them here.

where U' , D' are three dimensional vectors of the up and down-quark fields, U , D are the corresponding mass eigenstates and $V_{L,R}^U$, $V_{L,R}^D$ are 3×3 unitary matrices. In terms of these matrices, two CKM-like mixing matrices can be defined:

$$\begin{aligned} U_L^{CKM} &= V_L^{U\dagger} V_L^D, \\ U_R^{CKM} &= V_R^{U\dagger} V_R^D. \end{aligned} \quad (6)$$

The model file defines the quark field doublets by ascribing the mixing completely to the $T_3 = -\frac{1}{2}$ isospin states. This can be seen for example in the definition of the left-handed quark doublet:

```
QL[sp1_,1,ff_,cc_] := Module[{sp2}, ProjM[sp1,sp2] uq[sp2,ff,cc]],
QL[sp1_,2,ff_,cc_] := Module[{sp2,ff2}, CKML[ff,ff2]
  ProjM[sp1,sp2] dq[sp2,ff2,cc]],
```

where `ProjM` denotes the left handed projection operator, `sp1` and `sp2` are spinor indices, `ff` and `ff2` are generation indices and `cc` is a color index. The above definition keeps the $+\frac{1}{2}$ isospin states as unmixed left handed projections of the up type quarks mass eigenstates `uq`, and assigns the product of the up type quarks and the down type quarks mixings (defined in Eq.(6) as U_L^{CKM} and in the model file as `CKML`) to the left handed projections of the down type quarks mass eigenstates, which are the $-\frac{1}{2}$ isospin states.

In the absence of explicit CP violation in the Higgs potential, the relation between the above two CKM-like matrices can be written as [4]:

$$U_R^{CKM} = W^U U_L^{CKM} W^D, \quad (7)$$

where W^U and W^D are diagonal 3×3 matrices with ± 1 as diagonal elements, defined by

$$\begin{aligned} V_R^U &= V_L^U W^U, \\ V_R^D &= V_L^D W^D. \end{aligned} \quad (8)$$

This definition is aimed to set the sign of each element in the up type and down type diagonal mass matrix, respectively, to be positive. Also, the nature of the LRSM (i.e., either manifest or quasi-manifest) is set by the choice of diagonal entries in W^U and W^D : setting all the entries to $+1$ leads to the MLRSM, while setting one or more entries to -1 leads to the QMLRSM.

The relation in Eq.(7) is written in the model file as

```
Value -> {CKMR[a_,b_] -> WU[a,a]*CKML[a,b]*WD[b,b]}
```

where the three matrices `CKML`, `WU` and `WD` are adjustable (see Table A.2.) In particular, the user can adjust the `CKML` matrix by setting the values of its elements, whereas setting the `CKMR` matrix elements is done in a non-direct manner following the above definition; setting `WU[i,i]` and `WD[j,j]` to $+1$ for every i, j leads to the MLRSM

where $U_R^{CKM} = U_L^{CKM}$, while setting at least one diagonal element of WU or WD to be negative leads to the QMLRS, where $(U_L^{CKM})_{ij} = \pm (U_R^{CKM})_{ij}$. The following example demonstrates the adjustment of the $(U_L^{CKM})_{td}$ matrix element, followed by setting the t quark related elements in U_R^{CKM} to have the opposite sign from their corresponding elements in U_L^{CKM} . In the last step the newly defined U_R^{CKM} elements are printed in **Mathematica**.⁵

Example 1.

1. Find the CKML block in the Internal parameters section of the model file, and set
CKML[3,1] -> 0.00862
2. Find the WU block in the Internal parameters section of the model file, and set
WU[3,3] -> -1
3. In order to see the effect of step 2 on the relevant U_R^{CKM} matrix elements (via the definition of CKMR in the model file) save the program, then open **Mathematica** and type

```
$FeynRulesPath = SetDirectory["~/Mathematica/FeynRules"];
<< FeynRules'
SetDirectory[$FeynRulesPath <> "/Models"];
LoadModel["LRSM.fr"]
NumericalValue[WU[3,3]]
Do[Print[NumericalValue[
  ExpandIndices[IParamList[[h, 2]],
  FlavorExpand -> {Generation}]]], {h, zz, zz+2}]
```

where zz is the index number of the CKMR[3,1] parameter in the internal parameters list (to see the list type IParamList).

The lepton fields are similarly rotated into their mass eigenstates as follows:

$$\text{Fermion fields } \left\{ \begin{array}{l} n'_R = V N_R \\ n'_L = V^* N_L \\ l'_{L,R} = V_{L,R}^l l_{L,R} \end{array} \right\} \text{ Mass Eigenstates} \quad (9)$$

where n' and l' are six and three dimensional vectors representing the neutrino and charged lepton states respectively, N, l are the corresponding mass eigenstates and $V, V_{L,R}^l$ are 6×6 and 3×3 unitary matrices, respectively. Using the following definition of the V matrix

$$V = \left(\begin{array}{c} V_L^{\nu*} \\ V_R^\nu \end{array} \right), \quad (10)$$

⁵Adjusting the model parameters can also be carried out in **CalcHep** after the model translation. This is done with the **CalcHep** editing interface, see the manual of Ref. [9] for details.

one can define two 6×3 matrices analogues to the CKM mixing matrices of the quark sector:

$$\begin{aligned} K_L &= V_L^{\nu\dagger} V_L^l, \\ K_R &= V_R^{\nu\dagger} V_R^l. \end{aligned} \quad (11)$$

However, unlike the quark mixing matrices, both K_L and K_R can be independently adjusted.⁶ The lepton sector comprises a QMLRSM control matrix, W^l , which is the analogue to the W^D matrix of the quark sector. It is defined by:

$$V_R^l = V_L^l W^l. \quad (12)$$

The adjustment of the W^l diagonal elements is done in a similar way to the adjustments of W^U and W^D (see discussion above).

The lepton fields are implemented in the model similar to the quark fields, as can be seen for example from the following definition of the left-handed lepton doublet:

```
LL[sp1_,1,ff_] := Module[{sp2,ff2}, Conjugate[KL[ff2,ff]]
    ProjM[sp1,sp2] Nl[sp2,ff2]],
LL[sp1_,2,ff_] := Module[{sp2}, ProjM[sp1,sp2] l[sp2,ff]] }.
```

There are, however, two distinctions between the above definition and the quark doublet definition mentioned earlier. The first is the mixing matrix K_L which replaces U_L^{CKM} and the second is that, in the lepton sector, the mixing is assigned only to the $T_3 = +\frac{1}{2}$ isospin states, as opposed to the quark sector where the mixing is assigned to the $T_3 = -\frac{1}{2}$ states.

3. The Higgs sector

The Higgs sector of the LRSM consists of multiplets which break the left-right symmetry into the $U(1)_Q$ observed symmetry, where the electromagnetic charge Q is defined by the modified Gell-Mann-Nishijima formula

$$Q = I_{3L} + I_{3R} + \frac{B - L}{2}. \quad (13)$$

The symmetry breaking is also required to be (partly) left-right symmetric by itself, in order to obtain Dirac mass terms for the fermions. This is accomplished by introducing the left-right symmetric Higgs bidoublet (the numbers in the parenthesis stand for its properties under $SU(3)_C$, $SU(2)_L$ and $SU(2)_R$ and for its quantum number under

⁶One should note however that there are additional constraints which should be imposed on the matrices K_L and K_R , e.g., $K_L^T K_R = K_R^T K_L = 0$, see [6].

$B - L$, respectively):

$$\phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \quad (1, 2, 2, 0) . \quad (14)$$

This bidoublet acquires the VEVs $\langle \phi_{1,2}^0 \rangle = k_{1,2}$ and generates the Dirac masses of the fermions via its couplings to the fermion bilinears $\bar{Q}_L Q_R$ and $\bar{Q}_R Q_L$. This by itself is not yet sufficient to break the left-right symmetry of Eq.(2) to $U(1)_Q$.⁷ Thus, as mentioned in the introduction, the model outlined here also employs (for the additional symmetry breaking) the two Higgs triplets:

$$\Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix} \quad \text{L : } (1, 3, 1, 2) \quad \text{R : } (1, 1, 3, 2) \quad (15)$$

with condensates $\langle \Delta_{L,R} \rangle = \nu_{L,R}$. This setup is sufficient in order to further break the symmetry to $U(1)_Q$. Consequently, the gauge invariant Yukawa interaction terms are:

$$L_Y = - \sum_{i,j} \left[\bar{L}_{iL} \left((h_L)_{ij} \phi + (\tilde{h}_L)_{ij} \tilde{\phi} \right) L_{jR} - \bar{Q}_{iL} \left((h_Q)_{ij} \phi + (\tilde{h}_Q)_{ij} \tilde{\phi} \right) Q_{jR} \right. \\ \left. - \bar{L}_{iL}^c \Sigma_R (h_M)_{ij} L_{jR} - \bar{L}_{iR}^c \Sigma_L (h_M)_{ij} L_{jL} \right] + h.c. \quad (16)$$

where $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$, $\Sigma_{L,R} = i\tau_2 \Delta_{L,R}$ and $h_Q, h_L, h_M, \tilde{h}_Q, \tilde{h}_L$ are 3×3 Yukawa matrices in flavor space. These matrices can be defined in terms of the mass matrices and the mixing matrices [4]. For example, the h_Q matrix is defined in the model file as

```
yQ[a_,b_] :> Module[{sp5,sp6}, Sqrt[2]/(k1^2-k2^2) (k1 yMU[a,b]
- k2 CKML[a,sp5] yD0[sp5,sp6] HC[CKMR[b,sp6]])]
```

where **sp5** and **sp6** are generation indices and **HC** denotes hermitian conjugation (the other symbols are defined in Table A.1).

The neutrino mass terms obtained from Eq.(16) are of Majorana type, and diagonalizing the neutrino mass matrix gives the see-saw relations (neglecting generation mixing),

$$M_\nu \approx \frac{M_l^2}{M_N} \quad \text{and} \quad M_N \approx \nu_R , \quad (17)$$

where M_l is the charged lepton Dirac mass, and $M_{\nu,N}$ are the light and heavy Majorana neutrino masses, respectively. Note that Eq.(17) relates the large scale of the right-

⁷The reason is that the $B - L$ quantum number attributed to the bidoublet is zero and, in addition, the fields which acquire the VEVs have no electric charge. Therefore, after symmetry breaking, the vacuum remain symmetric under $U(1)_{B-L} \times U(1)_Q$ instead of just $U(1)_Q$ (see also Ref.[11]).

handed triplet VEV, ν_R , to the small (Majorana) masses of the observed neutrinos, i.e. $M_\nu \ll M_l \ll M_N$.

Employing the covariant derivative in order to maintain gauge invariance, the Higgs kinetic terms are written as

$$L_{Higgs}^{kin} = Tr \left[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L) \right] + Tr \left[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R) \right] + Tr \left[(D_\mu \phi)^\dagger (D^\mu \phi) \right] , \quad (18)$$

where

$$\begin{aligned} D_\mu \phi &= \partial_\mu \phi - ig_L \vec{W}_{L\mu} \frac{\vec{\tau}}{2} \phi + ig_R \phi \frac{\vec{\tau}}{2} \vec{W}_{R\mu} , \\ D_\mu \Delta_{L,R} &= \partial_\mu \Delta_{L,R} - ig_{L,R} \left[\frac{\vec{\tau}}{2} \vec{W}_{L,R\mu} \Delta_{L,R} \right] - ig' B_\mu \Delta_{L,R} . \end{aligned} \quad (19)$$

The most general scalar potential which is invariant under the left-right symmetry of the Higgs multiplets, i.e.

$$\Delta_L \leftrightarrow \Delta_R, \quad \phi \leftrightarrow \phi^\dagger, \quad (20)$$

is

$$\begin{aligned} V(\phi, \Delta_L, \Delta_R) &= \\ &- \mu_1^2 (Tr [\phi^\dagger \phi]) - \mu_2^2 \left(Tr [\tilde{\phi} \phi^\dagger] + \left(Tr [\tilde{\phi}^\dagger \phi] \right) \right) - \mu_3^2 \left(Tr [\Delta_L \Delta_L^\dagger] + Tr [\Delta_L \Delta_L^\dagger] \right) \\ &+ \lambda_1 \left((Tr [\phi \phi^\dagger])^2 \right) + \lambda_2 \left(\left(Tr [\tilde{\phi} \phi^\dagger] \right) + \left(Tr [\tilde{\phi}^\dagger \phi] \right)^2 \right) + \lambda_3 \left(Tr [\tilde{\phi} \phi^\dagger] Tr [\tilde{\phi}^\dagger \phi] \right) \\ &+ \lambda_4 \left(Tr [\phi \phi^\dagger] \left(Tr [\tilde{\phi} \phi^\dagger] + Tr [\tilde{\phi}^\dagger \phi] \right) \right) \\ &+ \rho_1 \left(\left(Tr [\Delta_L \Delta_L^\dagger] \right)^2 + \left(Tr [\Delta_R \Delta_R^\dagger] \right)^2 \right) \\ &+ \rho_2 \left(Tr [\Delta_L \Delta_L] Tr [\Delta_L^\dagger \Delta_L^\dagger] + Tr [\Delta_R \Delta_R] Tr [\Delta_R^\dagger \Delta_R^\dagger] \right) \\ &+ \rho_3 \left(Tr [\Delta_L \Delta_L^\dagger] Tr [\Delta_R \Delta_R^\dagger] \right) \\ &+ \rho_4 \left(Tr [\Delta_L \Delta_L] Tr [\Delta_R^\dagger \Delta_R^\dagger] + Tr [\Delta_L \Delta_L] Tr [\Delta_R^\dagger \Delta_R^\dagger] \right) \\ &+ \alpha_1 \left(Tr [\phi \phi^\dagger] \left(Tr [\Delta_L \Delta_L^\dagger] + Tr [\Delta_R \Delta_R^\dagger] \right) \right) \\ &+ \alpha_2 \left(Tr [\phi \tilde{\phi}^\dagger] Tr [\Delta_R \Delta_R^\dagger] + Tr [\phi^\dagger \tilde{\phi}] Tr [\Delta_L \Delta_L^\dagger] \right) \\ &+ \alpha_2^* \left(Tr [\phi^\dagger \tilde{\phi}] Tr [\Delta_R \Delta_R^\dagger] + Tr [\tilde{\phi}^\dagger \phi] Tr [\Delta_L \Delta_L^\dagger] \right) \\ &+ \alpha_3 \left(Tr [\phi \phi^\dagger \Delta_L \Delta_L^\dagger] + Tr [\phi^\dagger \phi \Delta_R \Delta_R^\dagger] \right) \end{aligned}$$

$$\begin{aligned}
& + \beta_1 \left(\text{Tr} \left[\phi \Delta_R \phi^\dagger \Delta_L^\dagger \right] + \text{Tr} \left[\phi^\dagger \Delta_L \phi \Delta_R^\dagger \right] \right) + \beta_2 \left(\text{Tr} \left[\tilde{\phi} \Delta_R \phi^\dagger \Delta_L^\dagger \right] + \text{Tr} \left[\tilde{\phi}^\dagger \Delta_L \phi \Delta_R^\dagger \right] \right) \\
& + \beta_3 \left(\text{Tr} \left[\phi \Delta_R \tilde{\phi}^\dagger \Delta_L^\dagger \right] + \text{Tr} \left[\phi^\dagger \Delta_L \tilde{\phi} \Delta_R^\dagger \right] \right), \tag{21}
\end{aligned}$$

where μ_i are mass parameters and $\lambda_i, \rho_i, \alpha_i, \beta_i$ are dimensionless couplings. With the exception of α_2 , all the parameters in the potential are real due to the Parity symmetry. Since in this work we assume that CP is explicitly conserved in the potential (in both the MLRSM and the QMLRSM), we also set α_2 to be real (see Ref.[6]). As for the mass parameters, by requiring that the potential has a minimum they can be expressed in terms of $\lambda_i, \rho_i, \alpha_i$ and β_i and of the Higgs VEVs [12]. The model file mass parameters `musq[1..3]` are therefore defined in terms of the (adjustable) couplings `lambda[1..4]`, `rho[1..4]` and `alpha[1..3]` (the zero valued β_i parameters are absent from the model file - see below) and of the Higgs VEVs `k1`, `k2` and `nuR` (see Table A.2 in [Appendix A](#) for the list of adjustable parameters and [Appendix B](#) for the relevant dependency equations).

As discussed in Ref.[4], the explicit CP conservation and minimization conditions impose a set of constraints on the model parameters, the first being that the Higgs bidoublet VEVs k_1 and k_2 must be real. A second constraint originates from two specific minimization conditions ($\partial V / \partial \nu_R = \partial V / \partial \nu_L = 0$) and is known as the 'VEV seesaw relation':

$$\beta_2 k_1^2 + \beta_1 k_1 k_2 + \beta_3 k_2^2 = (2\rho_1 - \rho_3) \nu_L \nu_R, \tag{22}$$

which gives

$$\nu_L = \gamma \frac{k_1^2 + k_2^2}{\nu_R}, \tag{23}$$

where

$$\gamma \equiv \frac{\beta_2 k_1^2 + \beta_1 k_1 k_2 + \beta_3 k_2^2}{(2\rho_1 - \rho_3)(k_1^2 + k_2^2)}. \tag{24}$$

Assuming that β_i and ρ_i are of order unity (i.e. not too large - to preserve unitarity, and not too small - to avoid fine tuning) implies that $\gamma \sim 1$. Since the light neutrino masses (which are proportional to ν_L via the Yukawa coupling) are bounded to be less than $\mathcal{O}(10)$ eV, ν_R has to be at least as large as $\mathcal{O}(10^7)$ GeV. This, in turn, leads to unobservably large masses for the additional Higgs and gauge bosons states (i.e., of order 10^7 GeV), unless the β_i are fine-tuned to reduce γ to about 10^{-6} and, thus, allow ν_R to be small enough, i.e., $\nu_R \sim \mathcal{O}(10^3)$ GeV. In this case, the new gauge-bosons and Higgs particles become accessible at the LHC (see mass formulae in [Appendix B](#)).

One possible way to avoid the (unwanted) fine-tuning of the Higgs couplings is to eliminate almost completely the VEV seesaw relation by setting some of the relevant parameters in the Higgs potential (in particular, the β parameters) to zero. This may

not be considered as fine tuning but, rather, as a possible consequence of some higher level exact symmetry (e.g., GUT or SUSY), which lies beyond the context of the LRSM [4].⁸ The only way in which the remainder of the VEV seesaw relation (i.e., Eq.(22) with $\beta_i = 0$) can be consistently (i.e., with observation) satisfied is by setting the VEV of the left-handed triplet, ν_L , to zero [10].⁹

Summarizing the above, the imposed constraints on the potential parameters and the Higgs VEVs are:

- The Higgs bidoublet VEVs k_1 and k_2 are real.
- The parameters β_i are set to zero.
- The Higgs left triplet VEV ν_L is set to zero.

The Higgs mass content is then determined by

$$\left. \frac{\partial^2}{\partial \phi_i \partial \phi_j} V \right|_{\phi_i = \phi_j = 0} = m_{i,j}^2 . \quad (25)$$

Using at first the minimization conditions of the potential and then the three assumptions/constraints listed above, one can calculate the physical Higgs masses. The expressions for the masses in terms of the free (adjustable) parameters in the Higgs potential are given in [Appendix B](#), for the case $\nu_R \gg k_{1,2}$, as required phenomenologically.¹⁰ These masses are implemented in the model file and the resulting mass values can be extracted using either *Mathematica* or *CalcHep*. The following example shows how to set the mass of H_2^0 ($= \sqrt{2\rho_1\nu_R^2}$, see [Appendix B](#)) in the model file and observe the result in *Mathematica*¹¹:

Example 2.

1. $M_{H_2^0}$ is represented by the symbol `MH02` in the model file, see [Table A.1](#).
2. Find the `MH02` block in the internal parameters section of the program, and observe that its value (which can be also found in [Appendix B](#)) is given by

`Value -> Sqrt[2*rho[1]*nuR^2],`

so that it depends on two adjustable parameters, `rho[1]` and `nuR` (the complete list and locations of adjustable parameters is given in [Table A.2](#)).

⁸Setting the β_i parameters to be small but not zero through mechanisms such as horizontal symmetry is discussed e.g., in Refs.[12, 13]. However here we follow the stricter case presented in Refs.[4, 6].

⁹The other option of setting $\nu_R = 0$ leads to $m_{W_2} \sim m_W$ in contrast with observation, and the option $2\rho_1 - \rho_3 = 0$ leads to massless Higgs bosons of the left-handed triplet, which is also ruled out experimentally[4].

¹⁰The heavy W_2 gauge-boson which depends on ν_R (and was not discovered yet) has to be significantly heavier than the SM W , since $m_W \propto \sqrt{k_1^2 + k_2^2}$.

¹¹See footnote 5.

3. Find the `rho` block in the *Internal parameters* section of the program and set a new value to `rho[1]`, e.g., `rho[1] -> 1/2`.
4. Find the `nuR` block in the *External parameters* section of the program and give it a new value, e.g. `Value -> 2000`.
5. Save the program and open **MATHEMATICA**, load the model into **MATHEMATICA** and type

`NumericalValue[MH02]`

4. Validation and output data

The model Implementation was validated as follows:

1. The Feynman rules for the model file were calculated in **Mathematica** using the command

`FeynmanRules[LSM]`

and were then matched with the Lagrangian terms in Ref.[6].

2. SM cross-sections calculated using the model file were compared and matched with their known values.¹² In particular, values of the relevant parameters were chosen to reproduce the SM case.
3. A number of processes calculated using the model file were compared numerically to theoretical predictions. The comparison was made for a range of processes and values of particle masses, for which the output of **CalcHep** was cross checked with independent calculations and with results from the literature, as follows (see Fig. 1):

- The 2-body decay $N_\mu \rightarrow Z \nu_\mu$,
Calculated using the 2-body decay width formula

$$\Gamma = \frac{1}{2P_{N_\mu}^0} \overline{|\mathcal{M}|^2} \frac{1}{8\pi} \lambda^{\frac{1}{2}} \left(1, \frac{m_Z^2}{m_{N_\mu}^2}, \frac{m_{\nu_\mu}^2}{m_{N_\mu}^2} \right), \quad (26)$$

where

$$\overline{|\mathcal{M}|^2}(N_\mu \rightarrow Z + \nu_\mu) \approx \frac{g_w^2 c_A^2}{4 \cos^2 \Theta_W} \left[(m_N^2 - m_Z^2) \left(2 + \frac{m_N^2}{m_Z^2} \right) \right], \quad (27)$$

c_A being the axial vector coupling of the Majorana neutrinos.

¹²It is important to note at this point that although the neutrinos in the model file are of Majorana nature (as opposed to the SM case), it gives similar neutral current cross-sections (numerically) as those of the SM, see also Ref.[14].

- The 2-body decay $Z_2 \rightarrow Z H$,
Calculated using Eq.(26) (assigning $m_N \rightarrow m_{Z_2}$ and $m_{\nu_\mu} \rightarrow m_H$) with

$$\overline{|\mathcal{M}|^2}(Z_2 \rightarrow Z + H) = \frac{|f_1|^2}{3} \left[2 + \frac{(m_{Z_2}^2 + m_Z^2 - m_H^2)^2}{4m_Z^2 m_{Z_2}^2} \right], \quad (28)$$

where

$$f_1 = -i \frac{g_w^2 \sqrt{k_1^2 + k_2^2}}{4} \left[\frac{\cos \phi \sqrt{1 - 2 \sin \theta_W}}{\cos \theta_W} + \cos \theta_W \sin \phi + \frac{\sin \theta_W^2 \sin \phi}{\cos \theta_W} \right] \\ \times \left[\cos \theta_W \cos \phi + \frac{\cos \phi \sin \theta_W^2}{\cos \theta_W} + \frac{\sqrt{1 - 2 \sin \theta_W} \sin \phi}{\cos \theta_W} \right]. \quad (29)$$

This calculation was also cross-checked numerically in the limit $\phi \rightarrow 0$ with the results given in [15].

- The 2-body decay $W_2 \rightarrow W H$,
Calculated similar to the decay $Z_2 \rightarrow Z H$ above, where f_1 in Eq.(29) is replaced by

$$f_2 = i \frac{g_w^2 k_1 k_2 (\cos \xi^2 - \sin \xi^2)}{\sqrt{k_1^2 + k_2^2}}. \quad (30)$$

This calculation was also cross-checked numerically with the results in [15].

- $u \bar{d} \rightarrow W_2 \rightarrow \mu^+ \nu_\mu$,
Cross-checked with the result in Ref. [16].
- The 2-body decay $W_2 \rightarrow W Z$,
Cross-checked with the result in Ref. [15].
- The 2-body decay $H_R^{++} \rightarrow \mu^+ \mu^+$,
Calculated using Eq.(26) with the squared amplitude

$$\overline{|\mathcal{M}|^2} = |f_3|^2 (M_{H_R^{++}}^2 - 2M_\mu^2), \quad (31)$$

where $f_3 = -i \frac{2}{\sqrt{2}\nu_R} (K_{R22}^2 M_{\nu_\mu} + K_{R52}^2 M_{N_\mu})$, and integrating over half the phase space due to the identical particles in the final state.

- The 2-body decay $H_1^+ \rightarrow \mu^+ \nu_\mu$,
Calculated using Eq.(26) with the squared amplitude

$$\overline{|\mathcal{M}|^2} = |f_4|^2 (M_{H_1^+}^2 - M_\mu^2), \quad (32)$$

where $f_4 = \frac{-i}{\nu_R} W_{22}^l (K_{R22}^2 K_{L22} M_{\nu_\mu} + K_{R52}^2 K_{L22} M_{N_\mu})$.

- The 2-body decay $H_1^0 \rightarrow \delta_R^{++} \delta_R^{--}$,

Calculated using Eq.(26) with the squared amplitude

$$|\overline{\mathcal{M}}|^2 = |f_5|^2, \quad (33)$$

$$\text{where } f_5 = \frac{-i}{\sqrt{k_1^2 + k_2^2}}((2k_1^2 - 2k_2^2)\alpha_2 + k_1 k_2 \alpha_3).$$

The theoretical results for these processes were compared to numerical calculations by **CalcHep** using the implemented model. The comparison is shown in Fig. 1. In the figure, the dots represent **CalcHep** numerical values while the curves correspond to the theory. An agreement of 0.01% was found. The specific numerical values chosen for the parameters involved in each process are given in [Appendix C](#).

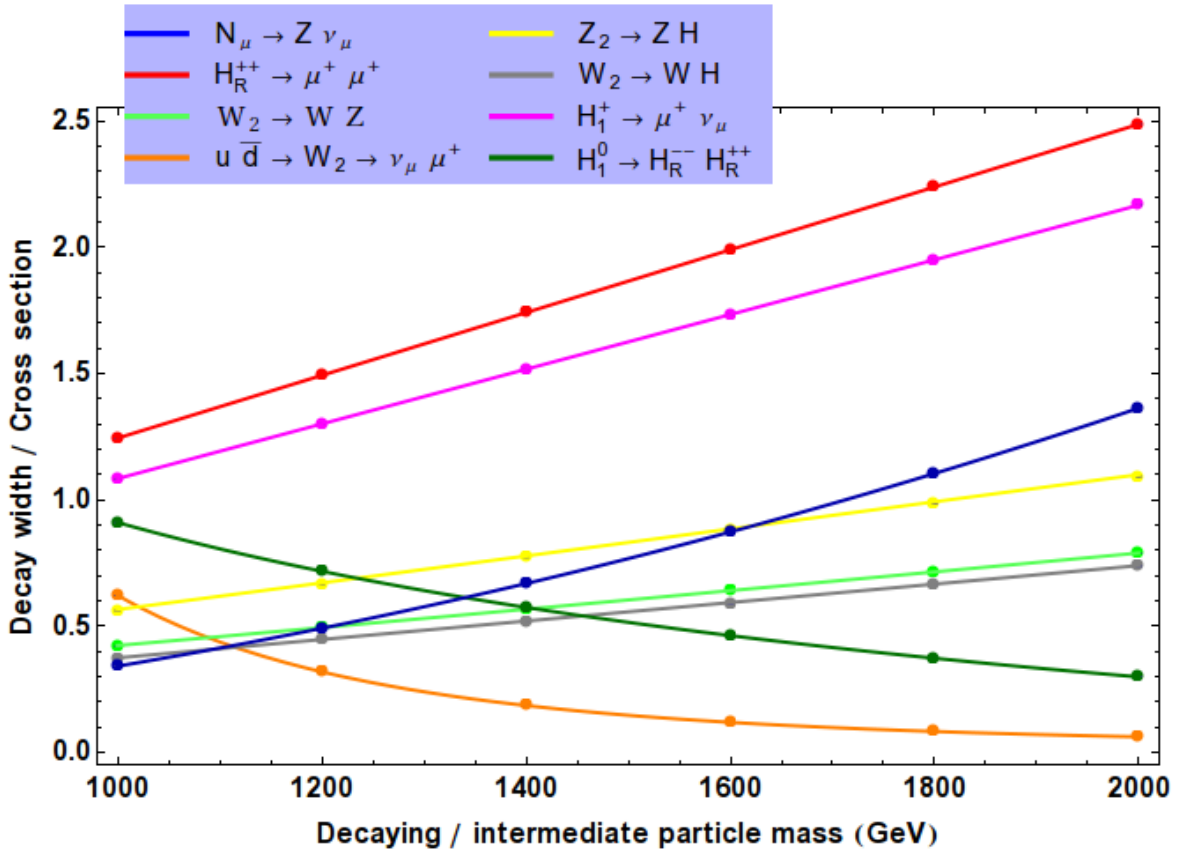


Figure 1: Cross-section and decay widths of LRSM (MLRSM) processes. The dots correspond to values calculated numerically by **CalcHep** while the curves correspond to the theoretical predictions. The measuring unit is GeV for the decays, except from $N_\mu \rightarrow Z \nu_\mu$ which is given in units of 10^{-8} GeV. The cross-section of $u \bar{d} \rightarrow W_2 \rightarrow \mu^+ \nu_\mu$ is given in units of 10^{-8} pb. See Table C.3 for the list of values used for the input parameters.

5. Summary

We described in this paper the left-right symmetric models MLRSM and QMLRSM and their implementation in **FeynRules**. The model was also tested with the **CalcHep** program and the **FeynArts** package. We presented the model Lagrangian and described how to apply its principles to the model file, also giving some simple examples. Finally, we described the validation tests performed for the model file. In light of the appealing new features that LRSM offer (to address some of the shortcomings of the SM), the MLRSM (QLRSM) implementation presented in this work can serve as an important tool for the search for new physics at the LHC era.

References

- [1] J.C. Pati and A. Salam, Phys. Rev. **D10**, 275 (1974);
R.N. Mohapatra and J.C. Pati, Phys. Rev. **D11**, 566 (1975);
G. Senjanović and R.N. Mohapatra, Phys. Rev. **D12**, 1502 (1975).
- [2] P. Minkowski, Phys. Lett. **B67**, 421 (1977);
R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980);
R.N. Mohapatra and G. Senjanović, Phys. Rev. **D23**, 165 (1981);
A. Maiezza, M. Nemevsek, F. Nesti, and G. Senjanović, Phys. Rev. **D82**, 055022 (2010), [arXiv:1005.5160](#) [hep-ph];
G. Senjanović, Int. J. Mod. Phys. **A26**, 1469 (2011), [arXiv:1012.4104](#) [hep-ph].
- [3] R. Alonso, M.B. Gavela, D. Hernandez, L. Merlo and S. Rigolin, [arXiv:1311.1724](#) [hep-ph] and references therein.
- [4] N.G. Deshpande, J.F. Gunion, B. Kayser and F.I. Olness, Phys. Rev. **D44**, 837 (1991).
- [5] A. Alloul, J. D'Hondt, K. De Causmaecker, B. Fuks and M. R. de Traubenberg, [arXiv:1301.5932](#) [hep-ph].
- [6] P. Duka, J. Gluza and M. Zralek, Annals Phys. **280**, 336 (2000), [arXiv:hep-ph/9910279](#).
- [7] N.D. Christensen, C. Duhr, Comp. Phys. Commun. **180**, 1614 (2009), [arXiv:0806.4194](#) [hep-ph];
A. Alloul, N.D. Christensen, C. Degrande, C. Duhr, B. Fuks, N.D. Christensen, C. Duhr, [arXiv:1310.1921](#) [hep-ph].
- [8] Wolfram Research, Inc., Mathematica, Version 7.0, Champaign, IL (2008).
- [9] A. Pukhov, [arXiv:hep-ph/0412191](#).
- [10] J.F. Gunion, J. Griford, A. Mendez, B. Kayser and F. Olness, Phys. Rev. **D40**, 1546 (1989).

- [11] R.N. Mohapatra and P.B. Pal *Massive Neutrinos in Physics and Astrophysics, Third Edition*, World Scientific Publishing Co. Pte. Ltd., 2004.
- [12] K. Kiers, M. Assis and A.A. Petrov, Phys. Rev. D **71**, 115015 (2005), [arXiv:hep-ph/0107121](#).
 *(We refer only to the case where the phases of the potential parameter α_2 and the Higgs vev k_2 are zero.)
- [13] C.-H. Lee, P.S. Bhupal Dev and R.N. Mohapatra, Phys. Rev. D **88**, 093010 (2013), [arXiv:1309.0774](#) [hep-ph].
- [14] B. Kayser (with Francoise Gibrat-Debu and Frederic Perrier) *The Physics of Massive Neutrinos*, World Scientific, 1989.
- [15] A. Deandrea, F. Feruglio and G.L Fogli, Nucl. Phys. **B402**, 3 (1993).
- [16] V.D. Barger, Roger J.N. Phillips, *Collider Physics (Updated Edition)*, Westview Press, 1996.
- [17] G. Barenboim, M. Gorbahn, U. Nierste and M. Raidal, Phys. Rev. D **65**, 095003 (2002).

Appendix A. Notation and adjustable parameters

The MLRSM input parameters and the corresponding symbols in the model file are collected in Table A.1 (following the notation in Ref.[6]). The list of adjustable parameters in the model file is given in Table A.2. Adjustment of other parameters in the file is of course possible, but consistency should be kept. For example, one can set the numeric value of MW2 instead of nuR, but then consistency of the model requires setting also $\text{nuR} = \text{MW2}/\text{MW} * \text{Sqrt}[(k_1^2 + k_2^2)/2]$.

Category	LRSM symbol	Feynrules model symbol
Fermion doublets (rotated - unphysical)	$Q_{iL}, Q_{iR}, L_{iL}, L_{iR},$ $(L^c)_{iL}, (L^c)_{iR} \ (i = 1, 2, 3)$	QL, QR, LL, LR LCL, LCR
Gauge boson fields (rotated - unphysical)	$W_{iL}, W_{iR}, B \ (i = 1, 2, 3)$	Wi, WRi, B
Particles names (physical states)	W, Z, A, g (SM Gauge bosons) W_2, Z_2 (Extra SM Gauge bosons) u, c, t (Up type quarks) d, s, b (Down type quarks) e, μ, τ (Charged leptons)	W, Z, A, G, W2, Z2 u, c, t (class name: uq) d, s, b (class name: dq) e, mu, ta (class name: l)
	ν_e, ν_μ, ν_τ (Light neutrinos) N_e, N_μ, N_τ (Heavy neutrinos)	NeL, NmL, NtL NeH, NmH, NtH (class name: N1)
	$H_0^0, H_1^0, H_2^0, H_3^0$ (Neutral Higgs scalars) A_1^0, A_2^0 (Neutral Higgs pseudoscalars) $H_1^\pm, H_2^\pm, \delta_L^{\pm\pm}, \delta_R^{\pm\pm}$ (Charged Higgs scalars) $\tilde{G}_1^0, \tilde{G}_2^0$ (Neutral Goldstone bosons) ¹³ G_L^\pm, G_R^\pm (Charged Goldstone bosons) ¹³	H, H01, H02, H03 A01, A02 HP1, HP2, HPPL, HPPR G01, G02 GPL, GPR
Particle masses	$M_{\text{Relevant Particle}}$	The letter M + Particle name
Decay widths	$\Gamma_{\text{Relevant Particle}}$	Either zero or the letter W + Particle name

¹³The model file uses the unitary gauge, so that all the Goldstone modes are omitted in the Feynman rules calculation.

Mixing Matrices	$U_L^{CKM}, U_R^{CKM}, K_L, K_R$	CKML, CKMR, KL, KR
Quasi Manifest Matrices	W^l, W^U, W^D	WL, WU, WD
Mixing angles	$\sin \Theta_W, \cos \Theta_W, \sin \xi$ $\cos \xi, \sin \phi, \cos \phi$	sw, cw, sxi cxi, sphi, cphi
Higgs VEV's	k_1, k_2, ν_L, ν_R	k1, k2, nuL, nuR
Higgs multiplets	$\phi, \tilde{\phi}, \Delta_{L,R}$	BD, BDtilde, LT, RT
Higgs multiplet field components	$\phi_{1,2}^0, \phi_{1,2}^\pm$ $\delta_{L,R}^0, \delta_{L,R}^\pm$ $\delta_{L,R}^{\pm\pm}$	Phi01, Phi02, PhiP1, PhiP2 H0L, H0R, HPL, HPR HDPL, HDPR
Parameters in the Potential	$\mu_{1..3}^2, \lambda_{1..4},$ $\rho_{1..4}, \alpha_{1..3}$	musq[1]..[3], lambda[1]..[4] rho[1]..[4], alpha[1]..[3]
Yukawa matrices	$h_q, \tilde{h}_q, h_l, \tilde{h}_l,$ h_M	yQ, yQtilde, yL, yLtilde, yHM1, yHM2 (both relate to h_M)
Diagonal mass matrices	$(M_U)_{\text{diag}}, (M_D)_{\text{diag}},$ $(M_l)_{\text{diag}}, (M_\nu)_{\text{diag}}$	yMU, yD0 yML, yNL
couplings	e, g, g', g_s	ee, gw, g1, gs

Table A.1: The MLRSM parameter names and the corresponding symbols in the FeynRules model file

LRSM parameter	FEYNRULES model symbol	Section in the file
Fermion masses	Mu, Mc, Mt Md, Ms, Mb Me, Mmu, Mta, MNeL, MMmL, MNtL MNeH, MMmH, MNtH	Particle classes / Fermions: physical fields
Gauge boson masses	MW, MZ	Particle classes / Gauge bosons: physical vector fields
Higgs VEV's	k1, nuR	Parameters / External Parameters
Pararameters in the Potential	lambda[1]..[4], rho[1]..[4], alpha[1]..[3]	Parameters / Internal Parameters Parameters / Internal Parameters Parameters / Internal Parameters
Mixing matrices	CKML, KL, KR	Parameters / Internal Parameters
Quasi manifest matrices	WU, WD, Wl	Parameters / Internal Parameters
Constants	aEWM1, Gf, aS, cabi	Parameters / External Parameters

Table A.2: The list of adjustable parameters in the model file.

Appendix B. Parameters, masses and mixing angles in the MLRSM

The non-adjustable parameters of the model file are defined by the expressions presented in this Appendix, which are based on the MLRSM (or QMLRSM) Lagrangian of Ref.[6]¹⁴. They consist of both adjustable and non-adjustable parameters which are listed in Tables A.1 and A.2. In particular, the following expressions and relations are given in the phenomenologically motivated approximation $\nu_R \gg k_{1,2}$:

$$\begin{aligned}
M_{W_1}^2 &\approx \frac{g^2}{4} k_+^2, \quad M_{W_2}^2 \approx \frac{g^2 \nu_R^2}{2}, \quad \tan 2\xi = -\frac{2k_1 k_2}{\nu_R^2}, \\
M_{Z_1}^2 &\approx \frac{g^2 k_+^2}{4 \cos^2 \Theta_W}, \quad M_{Z_2}^2 \approx \frac{\nu_R^2 g^2 \cos^2 \Theta_W}{\cos 2\Theta_W}, \quad \sin 2\phi \approx -\frac{k_+^2 (\cos 2\Theta_W)^{\frac{3}{2}}}{2\nu_R^2 \cos^4 \Theta_W} \\
M_{H_0^2}^2 &\approx 2k_+^2 \left(\lambda_1 + \frac{4k_1^2 k_2^2}{k_+^4} (2\lambda_1 + \lambda_3) + 2\lambda_4 \frac{2k_1 k_2}{k_+^2} \right), \\
M_{H_1^0}^2 &\approx \frac{\alpha_3 \nu_R^2}{2} \frac{k_+^2}{k_-^2}, \quad M_{H_2^0}^2 \approx 2\rho_1 \nu_R^2, \quad M_{H_3^0}^2 = \frac{1}{2} \nu_R^2 (\rho_3 - 2\rho_1), \\
M_{A_1^0}^2 &= \frac{\alpha_3 \nu_R^2}{2} \frac{k_+^2}{k_-^2}, \quad M_{A_2^0}^2 = \frac{1}{2} \nu_R^2 (\rho_3 - 2\rho_1), \\
M_{H_1^\pm}^2 &= \frac{1}{4} (\alpha_3 k_-^2) + \frac{1}{2} \nu_R^2 (\rho_3 - 2\rho_1), \quad M_{H_2^\pm}^2 = \frac{1}{4} \alpha_3 \left(k_-^2 + 2 \frac{k_+^2}{k_-^2} \nu_R^2 \right), \\
M_{\delta_L^{\pm\pm}}^2 &= \frac{1}{2} (\alpha_3 (k_1^2 - k_2^2) + \nu_R^2 (\rho_3 - 2\rho_1)), \quad M_{\delta^{\pm\pm}}^2 = \frac{1}{2} (\alpha_3 (k_1^2 - k_2^2) + 4\nu_R^2 \rho_2), \\
\mu_1^2 &\approx \nu_R^2 \left(\frac{\alpha_1}{2} - \frac{\alpha_3 k_2^2}{2(k_1^2 - k_2^2)} \right), \quad \mu_2^2 \approx \nu_R^2 \left(\frac{\alpha_2}{2} + \frac{\alpha_3 k_1 k_2}{4(k_1^2 - k_2^2)} \right), \quad \mu_3^2 \approx \rho_1 \nu_R^2, \quad (\text{B.1})
\end{aligned}$$

where $k_\pm \equiv \sqrt{k_1^2 \pm k_2^2}$ and $\lambda_i, \rho_i, \alpha_i$ are couplings in the Higgs potential (see Eq.(21)).

¹⁴The last three expressions also appear in Refs.[4, 17].

Appendix C. Adjusted values in the validation processes

The adjusted values which were used as input in each validation process are collected in Table C.3:

Process	Values of adjusted parameters
$N_\mu \rightarrow Z \nu_\mu$	$M_{\nu_\mu} = 10^{-8} \text{ GeV}$, $\nu_R = 3000 \text{ GeV}$.
$Z_2 \rightarrow Z H$	ν_R adjusted to give the relevant M_{Z_2} (see Appendix B).
$W_2 \rightarrow W H$	ν_R adjusted to give the relevant M_{W_2} (see Appendix B).
$W_2 \rightarrow W Z$	ν_R adjusted to give the relevant M_{W_2} (see Appendix B).
$u \bar{d} \rightarrow W_2 \rightarrow \mu^+ \nu_\mu$	$M_{N_\mu} = 500 \text{ GeV}$, $M_{\nu_\mu} = 10^{-8} \text{ GeV}$, $\Gamma_{W_2} = 20 \text{ GeV}$, $s = 4000 \text{ GeV}^2$, ν_R adjusted to give the relevant M_{W_2} (see Appendix B).
$H_R^{++} \rightarrow \mu^+ \mu^+$	$\nu_R = 2000 \text{ GeV}$, $M_{N_\mu} = 500 \text{ GeV}$, $M_{\nu_\mu} = 10^{-8} \text{ GeV}$, $\alpha_3 = 0.5$ ρ_2 adjusted to give the relevant $M_{H_R^{++}}$ (see Appendix B).
$H_1^+ \rightarrow \mu^+ \nu_\mu$	$\nu_R = 3000 \text{ GeV}$, $M_{N_\mu} = 700 \text{ GeV}$, $M_{\nu_\mu} = 10^{-8} \text{ GeV}$, $\alpha_3 = 0.5$, $\rho_1 = 0.3$, ρ_2 adjusted to give the relevant $M_{H_R^{++}}$ (see Appendix B).
$H_1^0 \rightarrow H_R^{++} H_R^{--}$	$\nu_R = 3000 \text{ GeV}$, $\rho_2 = 0.00005$, α_3 adjusted to give the relevant $M_{H_1^0}$ (see Appendix B).

Table C.3: Adjusted values of the parameters which were used in the validation processes.